Unit 3 Selected Answers

7.1

1. $x \approx 12.3$ 7. $x \approx 29.7°$ 13. $\alpha = 13°, a = 5, \beta = 17°, b \approx 6.5, \gamma = 150°, c \approx 11.11$

20. $\alpha = 42°, a = 17, \beta \approx 67.66°, b = 23.5, \gamma \approx 70.34°, c \approx 23.93$
or $\alpha = 42°, a = 17, \beta \approx 112.34°, b = 23.5, \gamma \approx 25.66°, c \approx 11.00$

27. $\alpha = 28.61°, a \approx 8.2, \beta \approx 102°, b = 16.75, \gamma \approx 49.39°, c = 13$

7.2

3. 57.1 7. a) 8.1 square units b) 377.1 square units c) 149 square units

9. 1204 feet 11. 1.4 miles 19. The Colonel is about 3193 feet from the campfire. Sarge is about 2525 feet from the campfire.

22. Jeff is about 371 feet from the nest.

7.3

1. 25.0 6. 70.7° 11. $\alpha = 104°, a \approx 49.41, \beta \approx 29.40°, b = 25, \gamma \approx 46.60°, c = 37$

15. $\alpha \approx 32.31°, a = 7, \beta \approx 49.58°, b = 10, \gamma \approx 98.21°, c = 13$

19. $\alpha \approx 22.62°, a = 5, \beta \approx 67.38°, b = 12, \gamma \approx 90°, c = 13$

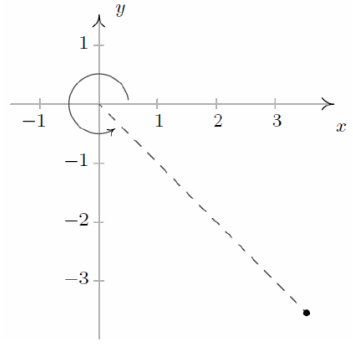
25. $\alpha \approx 3°, a \approx 29.72, \beta \approx 7°, b \approx 69.2, \gamma \approx 170°, c = 98.6$

28. 1.41 34. The diameter of the crater is about 5.22 miles.

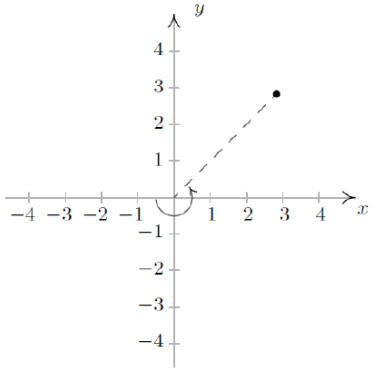
38. It is about 4.5 miles from port and its heading to port is $S47°W$.

8.1

2. $\left(5, \frac{7\pi}{4}\right), \left(-5, \frac{3\pi}{4}\right), \left(5, -\frac{\pi}{4}\right), \left(5, \frac{15\pi}{4}\right)$



10. $\left(-4, \frac{5\pi}{4}\right), \left(-4, \frac{13\pi}{4}\right), \left(4, -\frac{7\pi}{4}\right), \left(4, \frac{9\pi}{4}\right)$



19. $\left(-\frac{11\sqrt{3}}{2},\frac{11}{2}\right)$ 27. $\left(\frac{6\sqrt{5}}{5},\frac{12\sqrt{5}}{5}\right)$ 36. $(5,12)$ 43. $\left(8,\frac{4\pi}{3}\right)$ 51. $\left(13,\pi+\arctan\left(\frac{12}{5}\right)\right)$

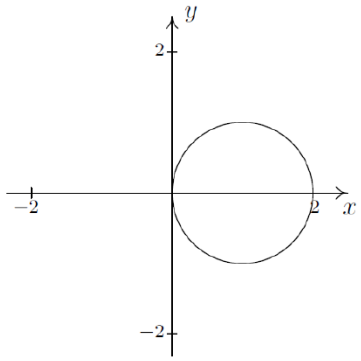
8.2

3. $r=7\csc(\theta)$ 11. $x=\dfrac{1}{\cos(\theta)-3\sin(\theta)}$ 17. $r=7\sin(\theta)$ 25. $y=-\sqrt{3}x$

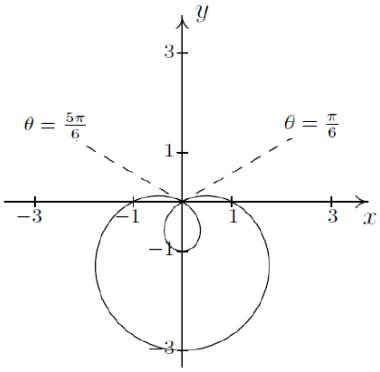
29. $5x^2+5y^2=x$ 35. $y=-\sqrt{5}$

8.3

2. Circle: $r=2\cos(\theta)$



14. Limacon: $r=1-2\sin(\theta)$



8.4

3. $z=6i=6\text{cis}\left(\frac{\pi}{2}\right)$, $\text{Re}(z)=0$, $\text{Im}(z)=6$, $|z|=6$

$\arg(z)=\left\{\frac{\pi}{2}+2\pi k \,\middle|\, k \text{ is an integer}\right\}$ and $\text{Arg}(z)=\frac{\pi}{2}$.

9. $z=-5i=5\text{cis}\left(\frac{3\pi}{2}\right)$, $\text{Re}(z)=0$, $\text{Im}(z)=-5$, $|z|=5$

$\arg(z)=\left\{\frac{3\pi}{2}+2\pi k \,\middle|\, k \text{ is an integer}\right\}$ and $\text{Arg}(z)=-\frac{\pi}{2}$.

15. $z=-7+24i=25\text{cis}\left(\pi-\arctan\left(\frac{24}{7}\right)\right)$, $\text{Re}(z)=-7$, $\text{Im}(z)=24$, $|z|=25$

$\arg(z)=\left\{\pi-\arctan\left(\frac{24}{7}\right)+2\pi k \,\middle|\, k \text{ is an integer}\right\}$ and $\text{Arg}(z)=\pi-\arctan\left(\frac{24}{7}\right)$.

25. $z=4\text{cis}\left(\frac{2\pi}{3}\right)=-2+2i\sqrt{3}$

8.5

In Exercises 1 – 12, we have that $z=-\frac{3\sqrt{3}}{2}+\frac{3}{2}i=3\text{cis}\left(\frac{5\pi}{6}\right)$ and $w=3\sqrt{2}-3i\sqrt{2}=6\text{cis}\left(-\frac{\pi}{4}\right)$ so we get the following.

1. $zw=18\text{cis}\left(\frac{7\pi}{12}\right)$

3. $\frac{w}{z}=2\text{cis}\left(\frac{11\pi}{12}\right)$

14. $\left(-\sqrt{3}-i\right)^3=-8i$

20. $\left(\frac{\sqrt{3}}{3}-\frac{1}{3}i\right)^4=-\frac{8}{81}-\frac{8i\sqrt{3}}{81}$

24. $(1-i)^8 = 16$

27. Since $z = 1 + i\sqrt{3} = 2\text{cis}\left(\frac{\pi}{3}\right)$ we have

$w_0 = \sqrt{2}\text{cis}\left(\frac{\pi}{6}\right) = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$ $\qquad$ $w_1 = \sqrt{2}\text{cis}\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$

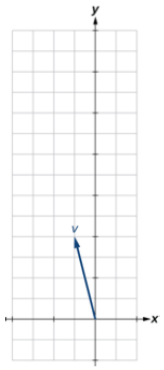
34. Since $z = -81 = 81\text{cis}(\pi)$ we have

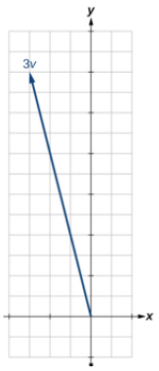
$w_0 = 3\text{cis}\left(\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$ $\qquad$ $w_1 = 3\text{cis}\left(\frac{3\pi}{4}\right) = -\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$

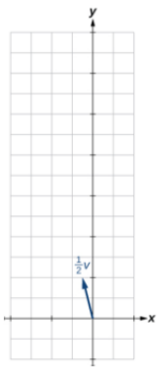
$w_2 = 3\text{cis}\left(\frac{5\pi}{4}\right) = -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$ $\qquad$ $w_3 = 3\text{cis}\left(\frac{7\pi}{4}\right) = \frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$

9.1

2. $\mathbf{v} = \langle -1, 4 \rangle$; $3\mathbf{v} = \langle -3, 12 \rangle$; $\frac{1}{2}\mathbf{v} = \left\langle -\frac{1}{2}, 2 \right\rangle$







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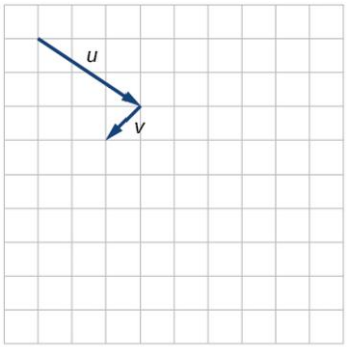
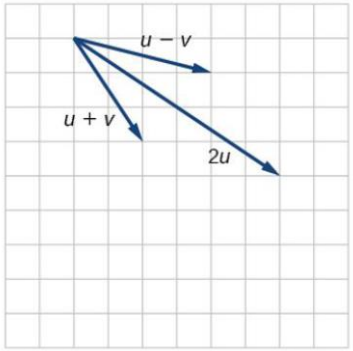




10. $\mathbf{v} + \mathbf{w} = \langle -12, 12 \rangle$, vector $\qquad$ $\mathbf{w} - 2\mathbf{v} = \langle 9, -60 \rangle$, vector $\qquad$ $\|\mathbf{v} + \mathbf{w}\| = 12\sqrt{2}$, scalar

$\|\mathbf{v}\| + \|\mathbf{w}\| = 38$, scalar $\qquad$ $\|\mathbf{v}\|\mathbf{w} - \|\mathbf{w}\|\mathbf{v} = \langle -34, -612 \rangle$, vector $\qquad$ $\|\mathbf{w}\|\mathbf{v} = \left\langle -\frac{91}{25}, \frac{312}{25} \right\rangle$, vector

18. $\langle 7,5\rangle$ 25. $\mathbf{v}=\langle -\sqrt{3},3\rangle$ 31. $\mathbf{v}=\langle 2,4\rangle$ 38. $\mathbf{v}=\langle -386.73,-230.08\rangle$

46. $\|\mathbf{v}\|=1,\quad \theta=240°$

9.2

3. $\mathbf{u}+(\mathbf{v}-\mathbf{w})=-5\mathbf{i}+3\mathbf{j}$ 7. $\frac{3}{5}\mathbf{i}+\frac{4}{5}\mathbf{j}$

12. $\mathbf{v}+\mathbf{w}=\mathbf{i}$, vector $\quad \mathbf{w}-2\mathbf{v}=-\frac{1}{2}\mathbf{i}-\frac{3}{2}\mathbf{j}$, vector $\quad \|\mathbf{v}+\mathbf{w}\|=1$, scalar

$\|\mathbf{v}\|+\|\mathbf{w}\|=\sqrt{2}$, scalar $\quad \|\mathbf{v}\|\mathbf{w}-\|\mathbf{w}\|\mathbf{v}=-\frac{\sqrt{2}}{2}\mathbf{j}$, vector $\quad \|\mathbf{w}\|\mathbf{v}=\frac{1}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}$, vector

19. She should fly at 83.46 miles per hour with a heading of N22.1°E.

23. The tension on the left hand cable is 285.317 lbs. and on the right hand cable is 92.705 lbs.

9.3

3. $\mathbf{u}\cdot\mathbf{v}=10$

7. $\mathbf{v}=\langle 1,\sqrt{3}\rangle$ and $\mathbf{w}=\langle 1,-\sqrt{3}\rangle$

$\mathbf{v}\cdot\mathbf{w}=-2,\ \theta=120°,\ \text{proj}_{\mathbf{w}}(\mathbf{v})=\left\langle -\frac{1}{2},\frac{\sqrt{3}}{2}\right\rangle,\ \mathbf{q}=\left\langle \frac{3}{2},\frac{\sqrt{3}}{2}\right\rangle$

17. $\mathbf{v}=3\mathbf{i}-\mathbf{j}$ and $\mathbf{w}=4\mathbf{j}$

$\mathbf{v}\cdot\mathbf{w}=-4,\ \theta\approx 108.43°,\ \text{proj}_{\mathbf{w}}(\mathbf{v})=\langle 0,-1\rangle,\ \mathbf{q}=\langle 3,0\rangle$

21. $\mathbf{v}=\left\langle \frac{1}{2},\frac{\sqrt{3}}{2}\right\rangle$ and $\mathbf{w}=\left\langle -\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right\rangle$

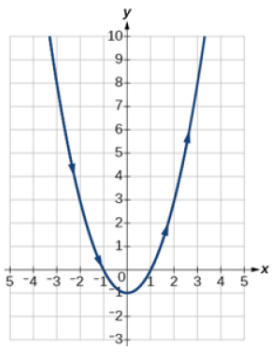
$\mathbf{v}\cdot\mathbf{w}=\frac{\sqrt{6}-\sqrt{2}}{4},\ \theta=75°,\ \text{proj}_{\mathbf{w}}(\mathbf{v})=\left\langle \frac{1-\sqrt{3}}{4},\frac{\sqrt{3}-1}{4}\right\rangle,\ \mathbf{q}=\left\langle \frac{1+\sqrt{3}}{4},\frac{1+\sqrt{3}}{4}\right\rangle$

27. $(13\text{ pounds})(25\text{ feet})\left(\cos\left(15°\right)\right)\approx 313.92\text{ foot-pounds}$

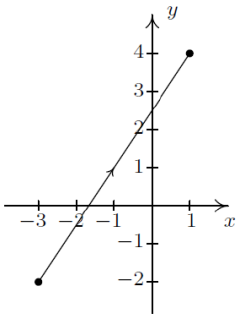
9.4 1.

$$\begin{cases} x(t)=t \\ y(t)=t^2-1 \end{cases}$$

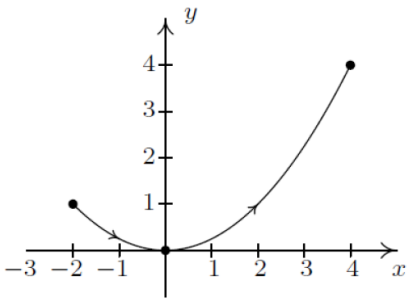
t	x	y
−3	−3	8
−2	−2	3
−1	−1	0
0	0	−1
1	1	0
2	2	3
3	3	8



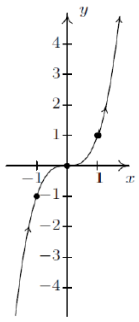
7. $\begin{cases} x = 4t - 3 \\ y = 6t - 2 \end{cases}$ for $0 \le t \le 1$



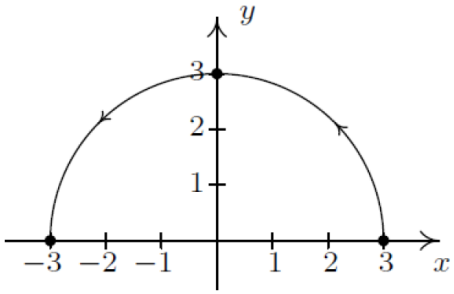
9. $\begin{cases} x = 2t \\ y = t^2 \end{cases}$ for $-1 \le t \le 2$



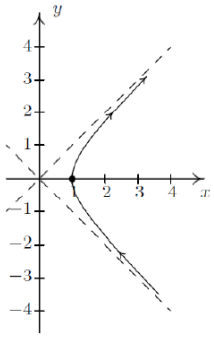
14. $\begin{cases} x = t \\ y = t^3 \end{cases}$ for $-\infty < t < \infty$



19. $\begin{cases} x = 3\cos(t) \\ y = 3\sin(t) \end{cases}$ for $0 \le t \le \pi$



25. $\begin{cases} x = \sec(t) \\ y = \tan(t) \end{cases}$ for $-\frac{\pi}{2} < t < \frac{\pi}{2}$



9.5

1. $y = -2 + 2x$

9. $x = \left(\frac{y}{2}\right)^3 - 1$

14. $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{6}\right)^2 = 1$

23. $\begin{cases} x = 2\cos(t) \\ y = 3\sin(t) \end{cases}$; Ellipse

27. $\begin{cases} x = 3 - 5t \\ y = -5 + 7t \end{cases}$ for $0 \le t \le 1$

33. $\begin{cases} x = 5\cos(t) \\ y = 5\sin(t) \end{cases}$ for $0 \le t < 2\pi$